

8300

GCSE MATHEMATICS

Foundation and Higher tier – Questions by Topic – Mark Scheme

Number

Finding prime numbers

v1.0

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

If a student uses a method which is not explicitly covered by the mark scheme the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied
B	Marks awarded independent of method.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
M dep	A method mark dependent on a previous method mark being awarded.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
oe	Or equivalent. Accept answers that are equivalent. e.g. accept 0.5 as well as $\frac{1}{2}$
[a, b]	Accept values between a and b inclusive.
3.14...	Allow answers which begin 3.14 e.g. 3.14, 3.142, 3.1416
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a student has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the student. In cases where there is no doubt that the answer has come from incorrect working then the student should be penalised.

Questions which ask students to show working

Instructions on marking will be given but usually marks are not awarded to students who show no working.

Questions which do not ask students to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Students often copy values from a question incorrectly. If the examiner thinks that the student has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Q1.

All conditions met:

- first number is prime
- second number is prime
- correctly evaluated
- even answer
- answer in range

if their product is incorrectly evaluated or missing, then 'even answer' and 'answer in range' refer to the correct product for their multiplication

B2 4 conditions met

B1 3 conditions met

B3

Additional Guidance

$2 \times 29 = 58$ (or $29 \times 2 = 58$) is the only fully correct solution

B3

Allow 50 to 60 inclusive for 'answer in range'

Award the best mark from boxes or in working for up to B2

The two prime numbers do not have to be different

[3]

Q2.

23 and 29

Either order

B1 one correct answer only

or one correct and one incorrect

or two correct and one incorrect

or any two prime numbers

B2

[2]

Q3.

(a) 16 or 9 seen

M1

7 (is prime)

A1

(b) Two different correct solutions

eg $x = 2, y = 1$ $x = 3, y = 2$

$x = 6, y = 5$ $x = 10, y = 9$

B2 for one correct solution

B1 for one correct trial

B3

[5]

Q4.

$51 = 3 \times 17$

*oe Multiplications **must** be shown
any order*

B1

$55 = 5 \times 11$

B1

$58 = 2 \times 29$

B1

[3]

Q5.

(a) 29

B1

(b) Any 3 of

B2 for 2 correct

$$33 = 3 \times 11 \quad 34 = 2 \times 17$$

B1 for 1 correct

$$35 = 5 \times 7 \quad 38 = 2 \times 19$$

SC1 two or more correct but outside range 30 to 40

B3

[4]

Q6.

(a) 3 7 13

B1 for 2 correct and 0 incorrect or for 3 correct and 1 incorrect

B2

(b) At least two correct substitutions evaluated correctly if answer not given
5, 11, 17, 23, 29, 35, ...

M1

($n =$) 6

or other correct values eg 11 or 13 or 16 or 20

A1

[4]

Q7.

5, 17, 37

B1 for each

SC2 – 4, 16, 36 on answer line

B3

Alternative

List of squares **or** list of primes

At least 3 (below 50) – allow 1 incorrect for every 4 correct

3 square numbers on answer line M1M0A0

M1

Adds 1 on to squares or subtracts 1 from primes

At least 3

M1Dep

5, 17, 37

All 3

SC2 – 4, 16, 36 on answer line

A1

[3]

Q8.

- (a) Correct set of four different prime numbers

B1

all numbers prime and the calculation correct, but with repeated numbers used

or

all numbers different and three of the four numbers prime and the calculation correct

or

at least four prime numbers identified with no incorrect numbers

or

at least five prime numbers identified with one incorrect number

B2

- (b) 2 is the only even prime number, so the sum must be even

oe Strand (ii)

Q1

2 is the only even prime number

or

(with 2 in) the sum would be even

or

even + odd + odd = even

or

2 can't be the answer (as it's the smallest prime number)

or

one or more correct numerical example(s) using 2, with no incorrect examples

Q2

[4]

Q9.

97

B1

[1]

Q10.

At least four square numbers seen or at least four cube numbers seen or 64 identified

M1

67 and prime

A1

[2]

Q11.

Identifies (11 and) 13 and 17 and 19

M1

Identifies 23 and 29

M1

1329 and 1723 and 1729 and 1923

*SC2 all 4 correct with one incorrect number treated as prime
or any 3 correct with no incorrect*

*SC1 any 3 correct with one incorrect number treated as prime
or any 2 correct with no incorrect*

A1

Additional Guidance

1329, 1723, 1729, 1923, 1327, 1927 (treats 27 as a prime number)

SC2

[3]

Q12.

29 and 31 with no other values *either order*

*B1 29 with at most one incorrect value
or 31 with at most one incorrect value*

B2

Additional Guidance

Ignore any values out of range for B1

1, 29, 31

B1

1, 23, 29

B1

[2]

Q13.

13 or 31 or 79 or 97 or 103 or any other prime whose digits add up to a square number

B1 any prime of 2 or more digits

*B1 any number of 2 or more digits whose digits add up to a
square number*

B2

[2]

Q14.

2 and 7

or 2 and 13

or 2 and 19

either order

*B1 any pair of **different** numbers chosen from 2, 3, 5, 7, 11, 13,
17, 19 eg 2 and 3 or 3 and 5*

B2

Additional Guidance

Mark the answer line but, if answer line blank, the pair of numbers must be
clearly selected for B2 or B1

List of prime numbers without selecting a pair

B0

[2]

Q15.

41, 43 and 47

B1

at least two of 41, 43 and 47 with at most one other number

B2

[2]

Q16.

23 or 29

implied by correct answer

B1

$$\frac{23}{125} (\times 100) \text{ or } \frac{29}{125} (\times 100)$$

or

$$\frac{\text{their number}}{125} (\times 100)$$

or

$$\text{their number} = \frac{125x}{100}$$

oe

their number can be any integer value

M1

18.4 or 23.2

or

correct evaluation of their number as a percentage of 125

ft B0M1

oe

their number must be an integer [20, 30] or any prime number

A1ft

Additional Guidance

18.4 or 23.2

B1M1A1

18.4 and 23.2

B1M1A1

23 or 29 must be clearly indicated as their prime number

Any integer [20, 30] used can score B0M1A1ft

eg $25 \div 125 \times 100$ with answer 20

B0M1A1ft

Any prime number used can score B0M1A1ft

eg $7 \div 125 \times 100$ with answer 5.6

B0M1A1ft

24% of 125 is 30 with answer 24

B0M1A1ft

29% of 125 is 36.25 (36.25 is not an integer)

B1M0A0ft

28% of 125 is 35 with answer 28 (35 is an integer out of range)

B0M1A0ft

28% of 125 is 35 scores M1 (35 is an integer)

25% of 125 is 31.25 scores M0 (31.25 is not an integer)

[3]

Q17.

At least two square numbers identified or a cube number identified
or 1 identified as the number

2 and prime

M1

A1

[2]

Q18.

Identifies 11 and 13 and 17 and 19

Identifies 23 and 29

M1

M1

1123, 1129, 1323, 1329, 1723, 1729, 1923, 1929

SC2

*all 8 correct with one incorrect number treated as prime
or any 6 correct with no incorrect*

SC1

*any 6 correct with one incorrect number treated as prime
or any 4 correct with no incorrect*

A1

[3]

Q19.

16 or 25 or 49

B1 any square of 2 digits

*B1 any number of 2 digits whose digits add up to a prime
number*

B2

[2]

Q20.

13, 11, 3

or

11,11,5

or

13,7,7

*B3 three primes that sum to 27 but cannot make a triangle (eg
17,7,3 or 19,3,5)*

*B2 three numbers that sum to 27 but only two are prime (eg
11,9,7)*

B1 three numbers that sum to 27 with 1 or 0 primes (eg 9,9,9)

B4

[4]

Q21.

27 in the box on the left side of calculation
accept 3^3 for 27 throughout

B1

Three different prime numbers in the boxes on the right side of calculation

M1

$$27 = 3 + 5 + 19$$

or

$$27 = 3 + 7 + 17$$

or

$$27 = 3 + 11 + 13$$

numbers in the boxes on the right side of calculation can be in any order

SC2

$$27 = 2 + 2 + 23$$

$$\text{or } 27 = 5 + 5 + 17$$

$$\text{or } 27 = 7 + 7 + 13$$

$$\text{or } 27 = 5 + 11 + 11$$

A1**Additional Guidance**

SC2 is for using a repeated prime number

$$27 = 3 + 5 + 17$$

B1M1A0

$$27 = 7 + 11 + 9$$

B1M0A0

$$27 = 1 + 3 + 23$$

B1M0A0

List of prime numbers with right side boxes empty or incorrect

M0**[3]****Q22.**

Meets all 4 criteria

B2 Their 2 values meet any 3 criteria

$$(x =) 64 \text{ and } (y =) 27$$

B1 Their 2 values meet any 2 criteria

or

B1 Reversed (x =) 27 and (y =) 64

B1 Reversed (x =) 64 and (y =) 81

$$(x =) 81 \text{ and } (y =) 64$$

SC2 (x =) 8^2 and (y =) 3^3 with no working

SC2 (x =) 9^2 and (y =) 4^3 with no working

SC1 a two digit square number subtract either 27 or 64

SC1 full list of two digit squares 16,25,36,49,64,81 with 27 and 64

B3**[3]**

Q23.

Any three of
 $a = 2, b = 5$
 $a = 7, b = 2$
 $a = 7, b = 11$
 $a = 11, b = 3$

B1 for each correct pair

SC1 if no other marks scored for:

$a = 3, b = 3$ (same numbers)

*$a = 3, b = 19$ oe (prime > 12) Listing all primes less than 12
 ie 2, 3, 5, 7, 11*

Listing the square numbers (1, 4,) 9, 16, 25

B3

[3]

Q24.

Any two sets of
 $a = 5, b = 3, c = 13$
 $a = 7, b = 11, c = 5$
 $a = 5, b = 2, c = 17$

B2 for any one set

B1 $a = 5, b = 5, c = 5$

or b and c prime and

a non-prime integer and

$$a = \sqrt{4b + c}$$

B3

[3]

Q25.

$a = 11, b = 23, c = 29$

B2 three trials using correct prime numbers

B1 all 4 prime numbers between 15 and 30 seen and at least one trial.

B3

[3]

Q26.

(a) $15ab$

B1 for $3a \times 5b$

B1 for partially simplified answer

B1 for $15 \times ab$

B2

Additional Guidance

Penalise further working, e.g. $3a \times 5b = 15ab = 3(5ab)$ gets B1

$15ba$

B2

$A = 15ab$

B2

$A(15ab)$

B2

$15ab \text{ cm}^2$

B2

$A(3a \times 5b)$

B1

$(3a)(5b)$

B1

$3a5b$

B1

$15(ab)$

B1

$3(5ab)$

B1

$ab15$

B1

$(15ab)^2$

B0

$15ab^2$

B0

$(3a \times 5b)^2$

B0

$3a \times 5b^2$

B0

(b) $315 \div 15$ or 21 seen

M1

7 and 3 in any order

SC1 for 15 and 21

or 9 and 35

A1

Additional Guidance

1 and 21 on the answer line

M1A0

[4]

Q27.

- (a) Substitutes and evaluates correctly to show that the answer is even

e.g.

$$5^2 + 3^2 = 34 \quad \text{or} \quad 3^2 + 5^2 = 34$$

$$25 + 9 = 34 \quad \text{or} \quad 9 + 25 = 34$$

$$7^2 + 3^2 = 58 \quad \text{or} \quad 3^2 + 7^2 = 58$$

$$49 + 9 = 58 \quad \text{or} \quad 9 + 49 = 58$$

$$7^2 + 5^2 = 74 \quad \text{or} \quad 5^2 + 7^2 = 74$$

$$49 + 25 = 74 \quad \text{or} \quad 25 + 49 = 74$$

Ignore fw

B1

Additional Guidance

One correct example required with or without incorrect examples

e.g. $2^2 + 3^2 = 13$, $5^2 + 3^2 = 34$

B1

- (b) Substitutes and evaluates correctly to show that the answer is odd

e.g.

$$3^2 + 2^2 = 13 \quad \text{or} \quad 2^2 + 3^2 = 13$$

$$9 + 4 = 13 \quad \text{or} \quad 4 + 9 = 13$$

$$5^2 + 2^2 = 29 \quad \text{or} \quad 2^2 + 5^2 = 29$$

$$25 + 4 = 29 \quad \text{or} \quad 4 + 25 = 29$$

$$7^2 + 2^2 = 53 \quad \text{or} \quad 2^2 + 7^2 = 53$$

$$49 + 4 = 53 \quad \text{or} \quad 4 + 49 = 53$$

Ignore fw

B1

Additional Guidance

One correct example required with or without incorrect examples

e.g. $2^2 + 3^2 = 13$, $5^2 + 3^2 = 34$

B1

[2]

Q28.

483 and 987

B2 for 3 or more digits correct

B1 for 2 digits correct

Allow commas, dashes, slashes etc between digit

B2 for total reverse eg 384 and 789

B3

[3]